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BUSINESS ADMINISTRATION

OPERATION RESEARCH I (120301451)

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Guidebook: **OPERATIONS RESEARCH;** Mishra, D.N.; Agarwal, S.K.; Lucknow, IND: Global Media, 2009.

TRAINING UNIT #2

2. Queuing Theory

2.1 Introduction

First time A.K. Erland, a Danish telephone Engineer, did original work on queuing theory. Erlang started his work in 1905 an attempt to determine the effects of fluctuating service demand (arrivals) on the utilization of automatic dialing equipment. In today's scenario a wide variety of seemingly diverse problems situations are recognized as being described by the general waiting line model.

In any queuing system, we have an input that arrives at some facility for service or processing and the time between the arrivals of individual's inputs at the service facility is commonly random in nature. For example, Doctor is a service facility and medical care is a service, ticket counter is a service facility and issue of ticket is service.

2.2 Queuing (Definition)

Queue or waiting lines stands for a number of customers waiting to be serviced. Queue does not include the customer being served. The process or system that performs the services to the customer is termed as service channel or service facility.

In general, we can say that a flow of customers from infinite or finite population towards the service facility forms a queue or waiting line on account of lack of capability to serve them all at time.

2.3 Queuing System

The formulation of queue is a common phenomenon which occurs whenever the current demand for a service exceeds the current capacity to provide that service. The qreues of a people may be seen at a cinema ticket windows, bus stop, reservation office, counters of super market etc: The person waiting in a queue or receiving the service is a called the customer and the person by whom he is serviced is called a server.

Thus the whole queue system is described as follows:

(a) The input (or arrival pattern)
(b) Queue (or waiting line)
(c) The server discipline (or queue discipline)
(d) The service mechanism (or service pattern)

Input ----------> Queuing Formation ----------> Service Mechanism ----------> Departure
Customer waiting

The basic Queuing system

(a) The input (or arrival pattern)

The input describes the pattern in which the customers arrive for service. Since the units for service in a random fashion therefore, their arrival pattern can be describe in terms of probabilities.

(b) Queue (or waiting line) formation

The units requiring service enter the queuing system on their arrival and join a "queue". A queue is called finite if the number of units in it is finite otherwise it is called infinite.

(c) The service discipline (or queue discipline):

The queue discipline is the manner in which the members in the queue are chosen for service. There are following queue disciplines:

(i) FCFS (First Come, First served):

According to this, the customers are served in the order of their arrival. This service discipline may be seen at a cinema ticket window, at a railway ticket window etc.

(ii) LCFS (Last come, First served):

According to this, the units (items) which come last are taken out (served) first.

(iii) SIRO (service in random order):

According to this, the customers are served in random order.

(iv) Service on some priority-procedure:

Some customers are served before the order without considering their ceder of arrival Le. some customers are served on priority basis.

(d) **The service mechanism (or service pattern):**

The service mechanism refers to

(i) The pattern according to which the customers are served.

(ii) Facilities given to the customer.

(iii) Single channel (the customer are served by one counter only).

(iv) Multi-channel (here the customer are served by several counters).

1. Single line Facility

2. Single queue Parallel Facilities Waiting line or queue

3. Multiple facilities

4. Series of facilities ~,--I _----' ~I L.._ _----' ~I'-__ ...J

5. Combination of facilities 194

3.4 Queuing Situation

There are various queuing situation which are shows is everyday life. In the queuing situation the customers are not only humans, but are non-human also.

Situation	Customers	Queue	Service Facility
Bank	Men and Women	Counter for cash withdrawal	ATM machines or counter clerk/ tellers
Airlines	Men and Women	Ticket counter	Service/ clerk/ cashier
Recruitment	Applicants	Arriving Candidate	Interviewers
Hospitals	Patient	Arriving Patient	Doctors

3.5 Queuing Model:

Kendall's notation for representing Queuing models

Generally, queuing model is represented by the following symbol form (a/b/c) :(d/e)

where

a = probability law for arrival time.
b = probability law according to which the customer are being served.
c = number of channels.
d = the maximum number allowed in the system (in service and waiting)
e = queue discipline.

Before dealing with carious models, we have to know certain symbols used.

	Poisson! exponential	M
	Only mean and variance known	G
1. Arrival and service process	Erlang	Ek
	Constant	D
	Normal	N
2. Number of server	One	1
	More than one	k
3. Queue discipline	First come first serve	FCFS
	Priority	PRI
	Random Selection	SIRO
4. Maximumqueue length	No limit	?
	Finite	n

Types of Queuing models

1. (M/M/1): (∞ / FCFS) Standard single server model

2. (M/M/k): (∞ / FCFS) Standard Multi server model

3. (M/Ek/1): (∞ / FCFS) Single Earlang service model

4. (M/G/1): (∞ / FCFS) Service time distribution unknown

5. (M/M/1): (∞ / PRI) Priority service, Single server

6. (M/M/1): (n / FCFS) Finite queue, single server

7. (M/M/k): (n/FCFS) Finite queue, Multi server

1. (M/MI1): (∞ / FCFS) (Birth and death Model):

Let us assume that the arrival rate is random and hence described by Poisson distribution. It is denoted by (λ). Service rate is assumed to follow negative exponential distribution and is denoted by (λ).

(i) Average number of customers in the system $L_s = \frac{\lambda}{(\mu - \lambda)}$

(ii) Expected queue length (Average number of customer in the system) $L_q = L_s - \frac{\lambda}{\mu}$

(iii) Average waiting time $\omega = \frac{1}{(\mu - \lambda)}$

(iv) Average waiting time in queue $W_q = \frac{\lambda}{\mu\,(\mu - \lambda)}$

(v) Average time in the system $W_s = \frac{1}{(\mu - \lambda)}$

(vi) Probability of an empty facility $P_{(0)} = 1 - \frac{\lambda}{\mu}$

(vii) A Probability of system being busy $P_{(w)} = 1 - P_{(0)} = \frac{\lambda}{\mu}$

(viii) Probability of being in the system longer than time t $P_{(T>t)} = e^{-(\mu - \lambda)t}$

(ix) Probability of customer not exceeding k in the system $P_{(n \geq k)} = p^k \; and \; (n>k) = p^{k+1}$

(x) Probability of exactly N $P_{(N)} = p^n(1-p)$

(xi) Traffic intensity $(p) = \frac{\lambda}{\mu}$

Relationship between Ls, Lq, Ws , Wq

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