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OPERATION RESEARCH I (120301451)

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TRAINING UNIT #1

1. PROGRAMME EVALUATION AND REVIEW TECHNIQUE AND CRITICAL PATH METHOD (PERT and CPM)

1.1 INTRODUCTION

Programme Evaluation and Review Technique (PERT) and Critical Path Method (CPM) are two techniques that are widely used in planning and scheduling the large projects. A project is a combination of various activities. For example, Construction of a house can be considered as a project. Similarly, conducting a public meeting may also be considered as a project. In the above examples, construction of a house includes various activities such as searching for a suitable site, arranging the finance, purchase of materials, digging the foundation, construction of superstructure etc. Conducting a meeting includes, printing of invitation cards, distribution of cards, arrangement of platform, chairs for audience etc. In planning and scheduling the activities of large sized projects, the two network techniques — PERT and CPM — are used conveniently to estimate and evaluate the project completion time and control the resources to see that the project is completed within the stipulated time and at minimum possible cost. Many managers, who use the PERT and CPM techniques, have claimed that these techniques drastically reduce the project completion time. But it is wrong to think that network analysis is a solution to all bad management problems. In the present chapter, let us discuss how PERT and CPM are used to schedule the projects.

Initially, projects were represented by milestone chart and bar chart. But they had little use in controlling the project activities. Bar chart simply represents each activity by bars of length equal to the time taken on a common time scale as shown in figure 15. l. This chart does not show interrelationship between activities. It is very difficult to show the progress of work in these charts. An improvement in bar charts is milestone chart. In milestone chart, key events of activities are identified and each activity is connected to its preceding and succeeding activities to show the logical relationship between activities. Here each key event is represented by a node (a circle) and arrows instead of bars represent activities, as shown in figure 15.2. The extension of milestone chart is PERT and CPM network methods.

Figure 15.l. Bar chart.

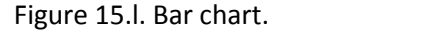

Figure 15.2. Milestone chart.

1.2 PERT AND CPM

In PERT and CPM the milestones are represented as events. Event or node is either starting of an activity or ending of an activity. Activity is represented by means of an arrow, which is resource consuming. Activity consumes resources like time, money and materials. Event will not consume any resource, but it simply represents either starting or ending of an activity. Event can also be represented by rectangles or triangles. When all activities and events in a project are connected logically and sequentially, they form a network, which is the basic document in network-based management. The basic steps for writing a network are:

(a) List out all the activities involved in a project. Say, for example, in building construction, the activities are:

(i) Site selection,
(ii) Arrangement of Finance,
(iii) Preparation of building plan,
(iv) Approval of plan by municipal authorities,
(v) Purchase of materials,
(vi) Digging of foundation,
(vii) Filling up of foundation,
(viii) Building superstructure,
(ix) Fixing up of doorframes and window frames,
(x) Roofing,
(xi) Plastering,
(xii) Flooring,
(xiii) Electricity and water fittings,
(xiv) Finishing.

(b) Once the activities are listed, they are arranged in sequential manner and in logical order. For example, foundation digging should come before foundation filling and so on.

(c) After arranging the activities in a logical sequence, their time is estimated and written against each activity. For example: Foundation digging: 10 days, or 1½ weeks.

(d) Some of the activities do not have any logical relationship, in such cases; we can start those activities simultaneously. For example, foundation digging and purchase of materials do not have any logical relationship. Hence both of them can be started simultaneously. Suppose foundation digging takes 10 days and purchase of materials takes 7 days, both of them can be finished in 10 days. And the successive activity, say foundation filling, which has logical relationship with both of the above, can be started after 10 days. Otherwise, foundation digging and purchase of materials are done one after the other; filling of foundation should be started after 17 days.

(e) Activities are added to the network, depending upon the logical relationship to complete the project network.

Some of the points to be remembered while drawing the network are

(a) There must be only one beginning and one end for the network, as shown in figure 15.3.

Figure 15. 3. Writing the network.

(b) Event number should be written inside the circle or node (or triangle/square/rectangle etc). Activity name should be capital alphabetical letters and would be written above the arrow. The time required for the activity should be written below the arrow as in figure 15. 4

Figure 15.4. Numbering and naming the activities.

(c) While writing network, see that activities should not cross each other. And arcs or loops as in figure 15.5 should not join Activities.

Figure 15.5. Crossing of activities not allowed.

(d) While writing network, looping should be avoided. This is to say that the network arrows should move in one direction, i.e. starting from the beginning should move towards the end, as in figure 15.6.

Figure 15. 6. Looping is not allowed.

(e) When two activities start at the same event and end at the same event, they should be shown by means of a dummy activity as in figure 15.7. Dummy activity is an activity, which simply shows the logical relationship and does not consume any resource. It should be represented by a dotted line as shown. In the figure, activities C and D start at the event 3 and end at event 4. C and D are shown in full lines, whereas the dummy activity is shown in dotted line.

Figure 15.7. Use of Dummy activity.

(e) When the event is written at the tail end of an arrow, it is known as tail event. If event is written on the head side of the arrow it is known as head event. A tail event may have any number of arrows (activities) emerging from it. This is to say that an event may be a tail event to any number of activities. Similarly, a head event may be a head event for any number of activities. This is to say that many activities may conclude at one event. This is shown in figure 15.8.

Figure 15.8. Tail event and Head event.

The academic differences between PERT network and CPM network are:

(i) PERT is event oriented and CPM is activity oriented. This is to say that while discussing about PERT network, we say that Activity 1-2, Activity 2-3 and so on. Or event 2 occurs after event 1 and event 5 occurs after event 3 and so on. While discussing CPM network, we say that Activity A follows activity B and activity C follows activity B and so on. Referring to the network shown in figure 9, we can discuss as under.

PERT way: Event 1 is the predecessor to event 2 or event 2 is the successor to event 1. Events 3 and 4 are successors to event 2 or event 2 is the predecessor to events 3 and 4.

CPM way: Activity 1-2 is the predecessor to Activities 2-3 and 2-4 or Activities 2-3 and 2-4 are the successors to activity 1-2.

(ii) PERT activities are probabilistic in nature. The time required to complete the PERT activity cannot be specified correctly. Because of uncertainties in carrying out the activity, the time cannot be specified correctly. Say, for example, if you ask a contractor how much time it takes to construct the house, he may answer you that it may take 5 to 6 months. This is because of his expectation of uncertainty in carrying out each one of the activities in the construction of the house. Another example is if somebody asks you how much time you require to reach railway station from your house, you may say that it may take 1 to 1½ hours. This is because you may think that you may not get a transport facility in time. Or on the way to station, you may come across certain work, which may cause delay in your journey from house to station. Hence PERT network is used when the activity times are probabilistic.

Figure 15.9. Logical relationship in PERT and CPM.

Figure 15.10. Three Time estimates.

There are three time estimates in PERT, they are:

(a) OPTIMISTIC TIME: Optimistic time is represented by t O . Here the estimator thinks that everything goes on well and he will not come across any sort of uncertainties and estimates lowest time as far as possible. He is optimistic in his thinking.

(b) PESSIMISTIC TIME: This is represented by t P . Here estimator thinks that everything goes wrong and expects all sorts of uncertainties and estimates highest possible time. He is pessimistic in his thinking.

(c) LIKELY TIME: This is represented by t_L . This time is in between optimistic and pessimistic times. Here the estimator expects he may come across some sort of uncertainties and many a time the things will go right.

So while estimating the time for a PERT activity, the estimator will give the three time estimates. When these three estimates are plotted on a graph, the probability distribution that we get is closely associated with Beta Distribution curve. For a Beta distribution curve as shown in figure 6.10, the characteristics are:

Standard deviation = $(t_P - t_O)/6 = \sigma$, $t_P - t_O$ is known as range

Variance = $\{(t_P - t_O)/6\}^2 = \sigma^2$

Expected Time or Average Time = $t_E = (t_O + 4t_L + t_P)/6$

These equations are very important in the calculation of PERT times. Hence the student has to remember these formulae.

Now let us see how to deal with the PERT problems.

(g) Numbering of events: Once the network is drawn the events are to be numbered. In PERT network, as the activities are given in terms of events, we may not experience difficulty. Best in case of CPM network, as the activities are specified by their name, is we have to number the events. For numbering of events, we use D.R. Fulkerson's rule.
As per this rule:
An initial event is an event, which has only outgoing arrows from it and no arrow enters it.
Number that event as 1.
Delete all arrows coming from event 1. This will create at least one more initial event.
Number these initial events as 2, 3 etc.
Delete all the outgoing arrows from the numbered element and which will create some more initial events. Number these events as discussed above.
Continue this until you reach the last event, which has only incoming arrows and no outgoing arrows.

While numbering, one should not use negative numbers and the initial event should not be assigned 'zero'. When the project is considerably large, at the time of execution of the project, the project manager may come to know that some of the activities have been forgotten and they are to be shown in the current network. In such cases, if we use skip numbering, it will be helpful. Skip numbering means, skipping of some numbers and these numbers may be made use to represent the events forgotten. We can skip off numbers like 5, 10, 15 etc. or 10, 20 and 30 or 2, 12, 22 etc. Another way of numbering the network is to start with 10 and the second event is 20 and so on. This is a better way of numbering the events.

1. **Updating the network:** In large project works, as the project progresses, we may come across situation like

(a) The time estimates made before may be wrong that a particular activity may take less or more time. And we may also sense that we have forgotten certain activities. In such cases, we have to update the project.

Leaving the executed activities, remaining activities may have to be modified and the remaining network is redrawn. This is known as updating the network.

2. **Resource leveling and Resource smoothing**: When we have to manage project with available resources, we have two options. First one is resource leveling. Here when the resources availability is less than the maximum resources required for an activity, then delay the job having largest float and divert the resources to critical activities. When two or more jobs compete for same resource, first try to allocate to an activity, which is of short

duration and next to the activity which having next highest duration. Here available resource is a constraint. The project duration time may increase during the process.

3. **Resource smoothing:** Here total project duration is maintained to the minimum level. By shifting the activities having floats the demand for resources are smoothened. Here main constraint is project duration time.

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